Stochastics of radioactive decay

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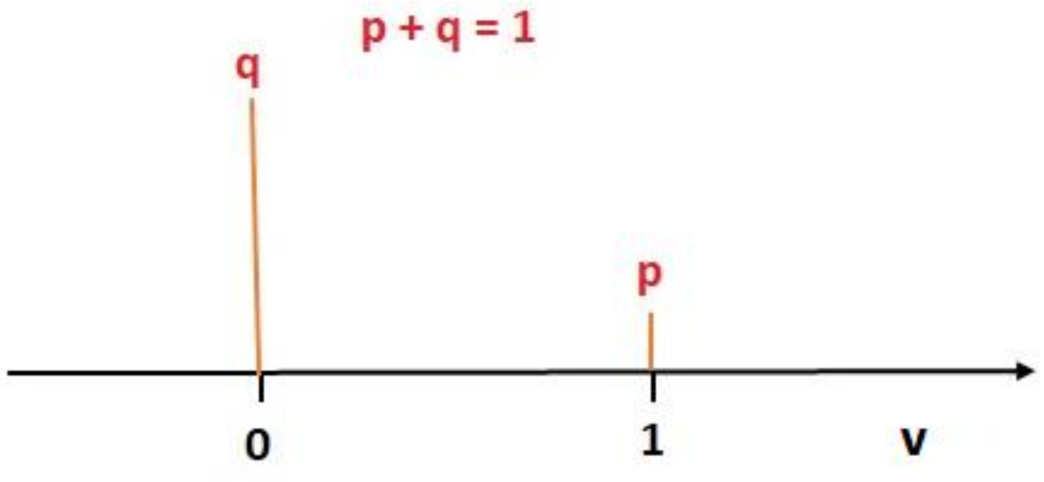

Binomial distribution

$P(\mathrm{v}) \in \{0,1\}$

$$p = 1 - \exp(-\lambda T) \qquad q = \exp(-\lambda T)$$

Rainwater and Wu, Nucleonics 1 (1947), 60-69.

$$n = N\,(1 - exp(-\lambda T))$$

$$r = \sum_{i=1}^{N} \mathrm{v}_i$$

$$N - n = N\,exp(-\lambda T))$$

$$r \in \{0, N\}$$

$$P(r) = \binom{N}{r} \mathrm{p}^r \mathrm{q}^{N-r}$$

$$= \binom{N}{r} (1 - e^{-\lambda T})^r (e^{-\lambda T})^{N-r}$$

$$E(\mathrm{v}) = 0\mathrm{q} + 1\mathrm{p} = \mathrm{p}.$$

$$Var(\mathrm{v}) = ((0 - E(\mathrm{v}))^2\mathrm{q} + (1 - E(\mathrm{v})^2\mathrm{p} = \mathrm{p}^2\mathrm{q} + (1 - \mathrm{p})^2\mathrm{p} = \mathrm{pq}.$$

The expectation of the sum of stochastic variables is the sum of the expectations of the variables.

$$E(r) = \sum_{i=1}^{N} E(\mathrm{v}_i) = N\mathrm{p} = n.$$

The variance of the sum of independant stochastic variables is the sum of the variances of the variables.

$$Var(r) = \sum_{i=1}^{N} Var(\mathrm{v}_i) = N\mathrm{p}\,\mathrm{q} = n\exp(-\lambda T).$$

$$\Delta n = \sqrt{n}\,\exp(-\lambda T/2).$$

^{40}K, $t_{½} = 1.26\ 10^{9}$ a,

photon 1.460 MeV in 10.72 % emissions.

10 g rock, 1 h measurement. Then $N = 3.96\ 10^{16}$.
We find $n = 2483$ emitted photons per 1 h and
$\Delta n = 49.8$ and $\Delta n/n = 2.0$ %.

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y = binopdf(x,N,p);          y=poisspdf(x,n);
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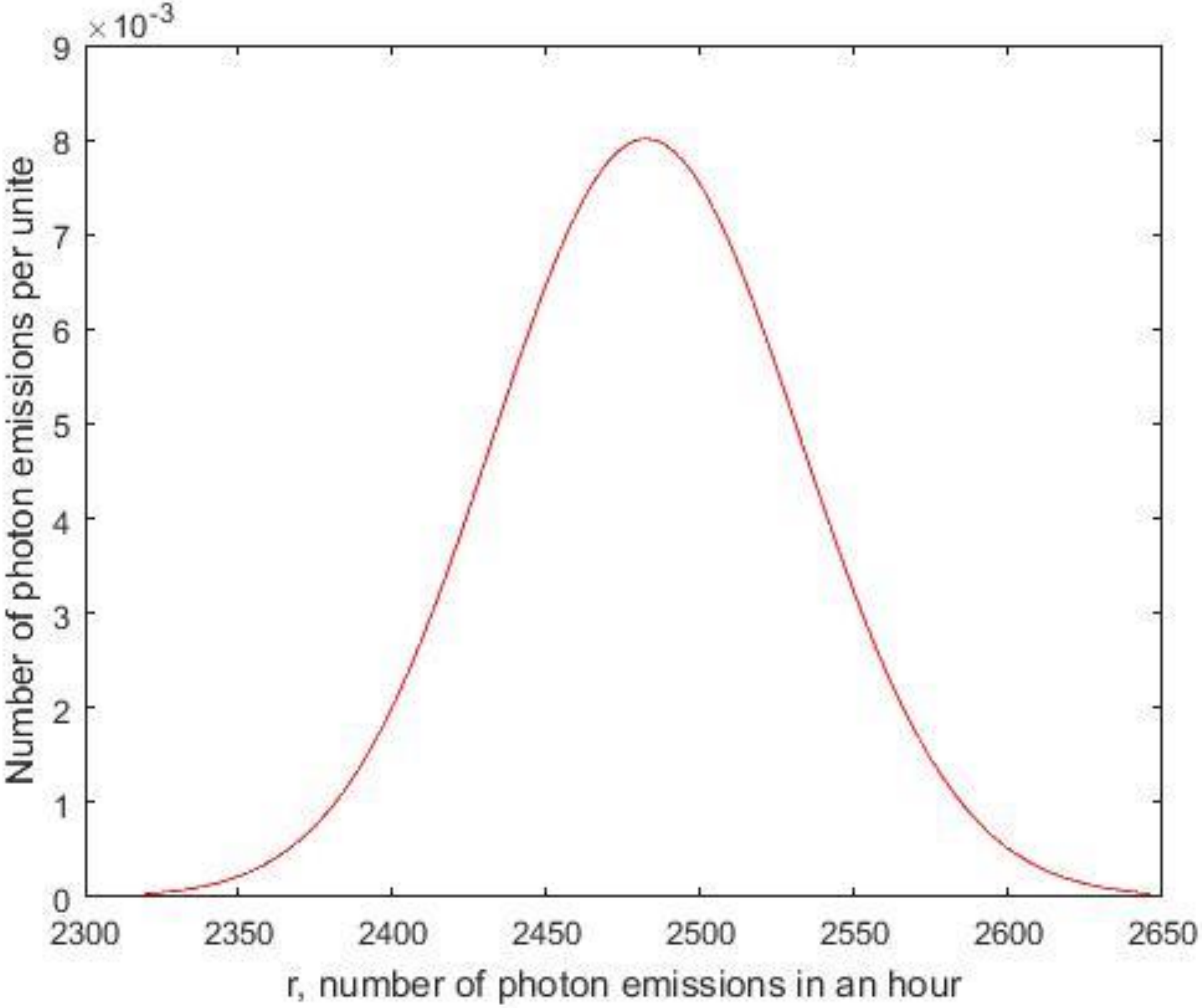
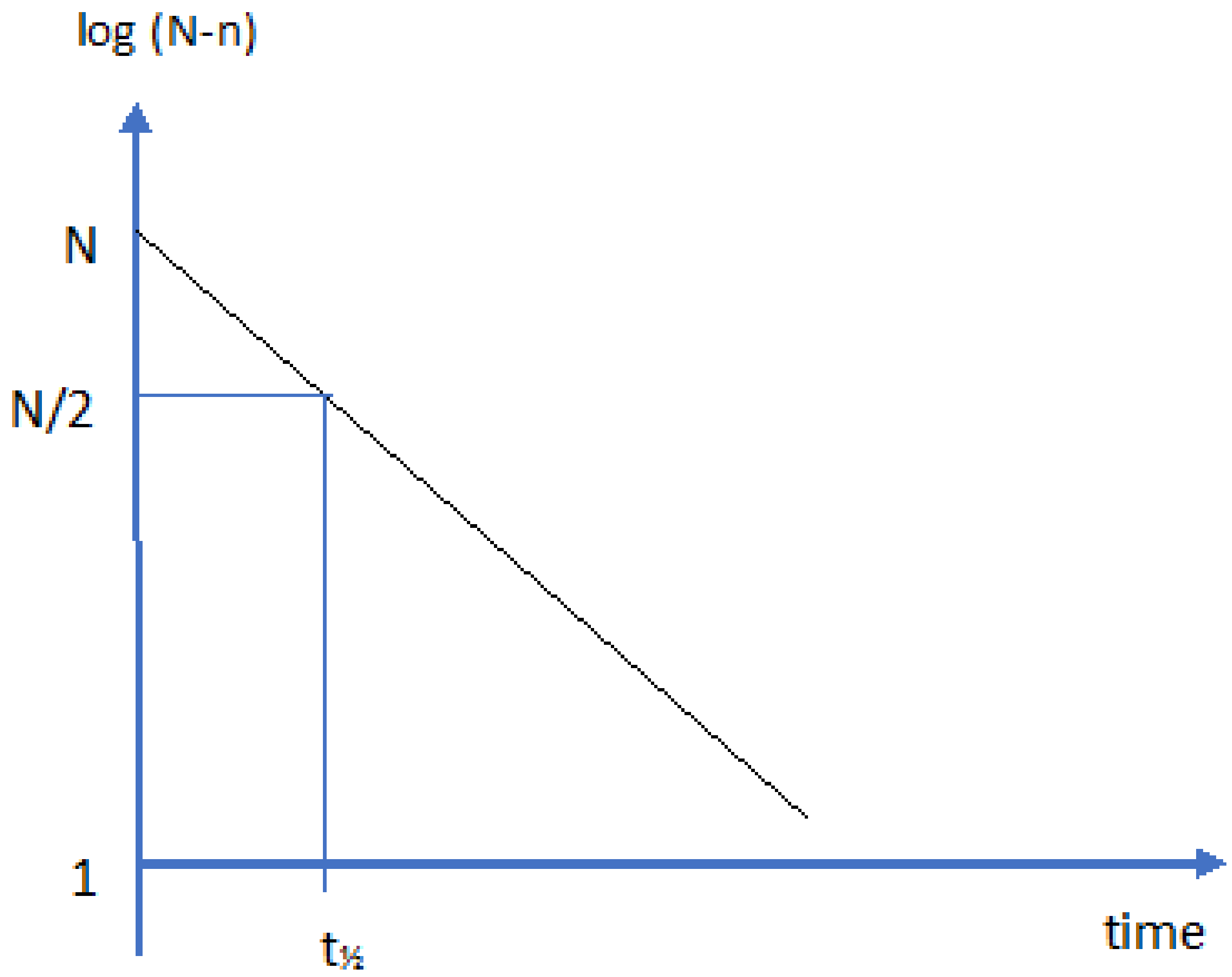
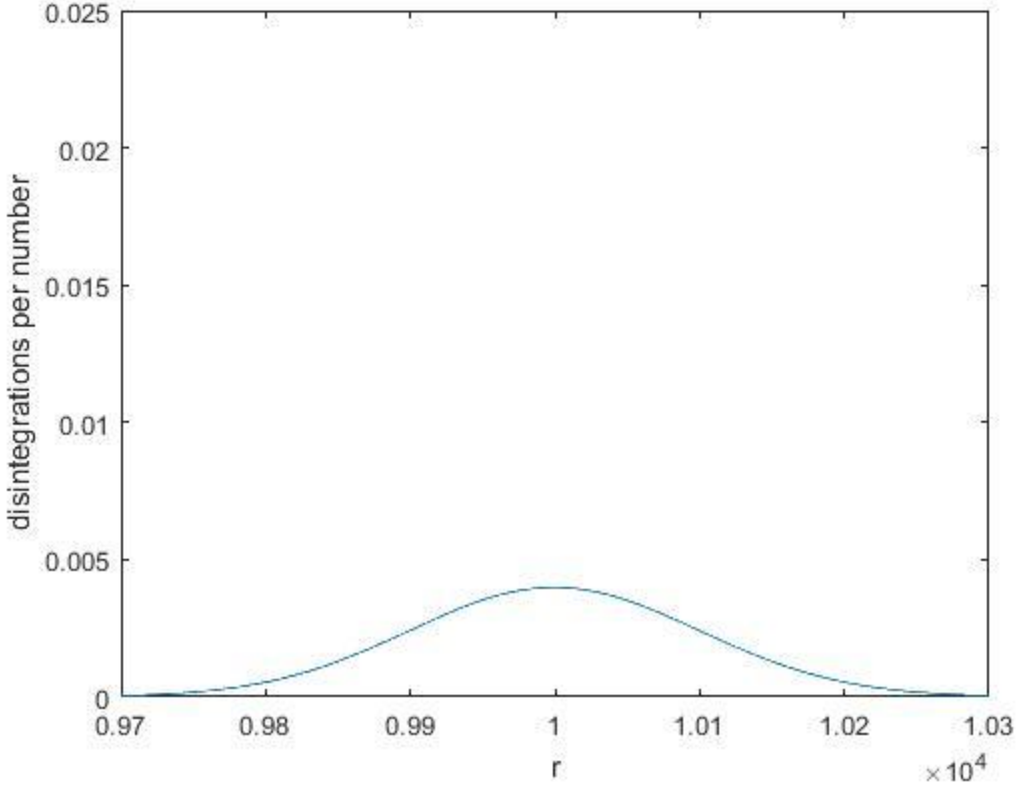


×10^-3
9
8
7
6
5
4
3
2
1
0
Number of photon emissions per unite
2300
2350
2400
2450
2500
2550
2600
2650
r, number of photon emissions in an hour

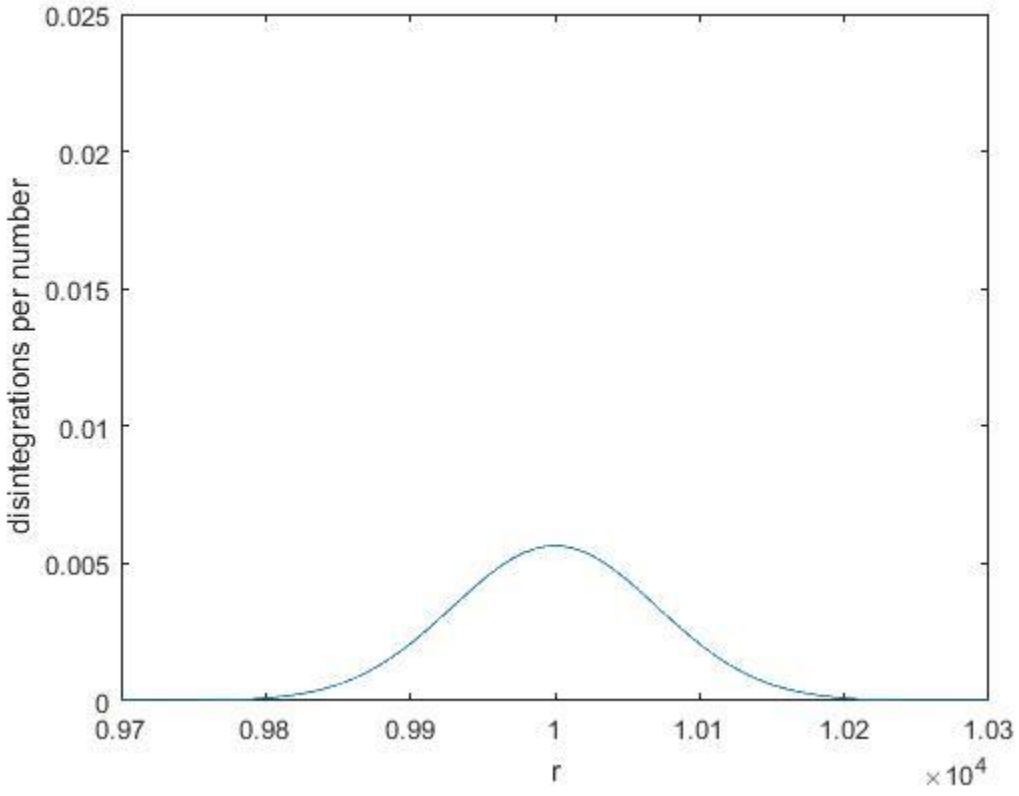


log (N-n)
N
N/2
1
t½
time

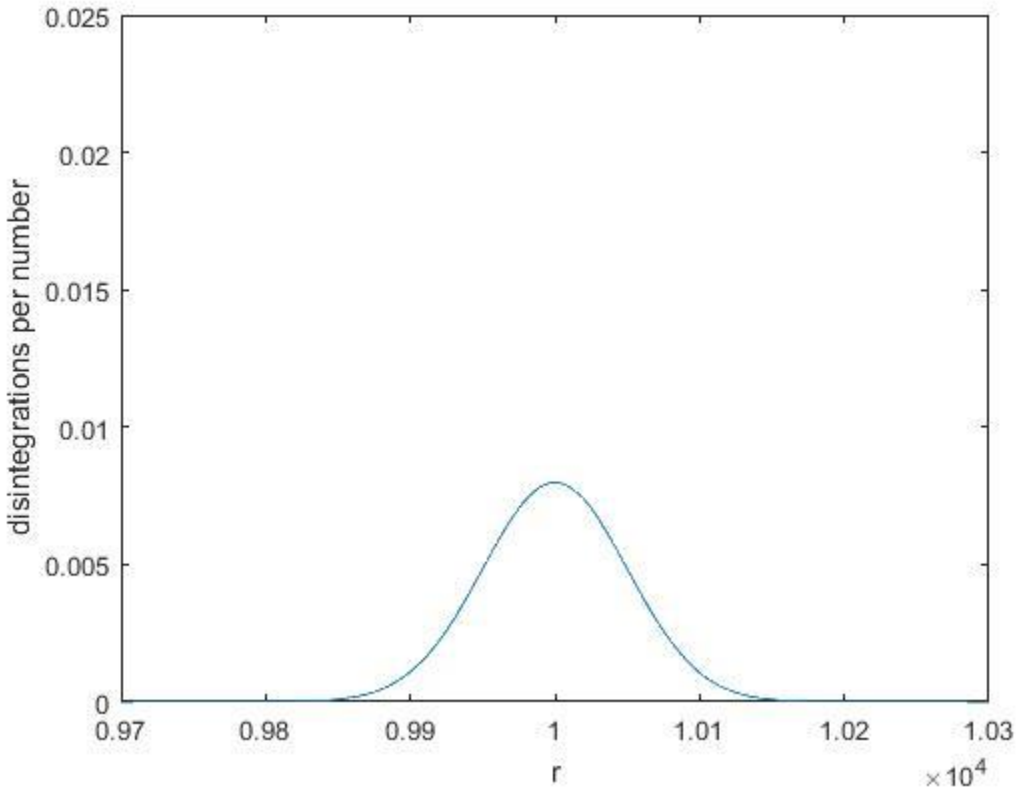
$T \ll t_{1/2}$


0.025
0.02
0.015
0.01
0.005
0
disintegrations per number
0.97
0.98
0.99
1
1.01
1.02
1.03
r
×10^4

T = $t_{½}$

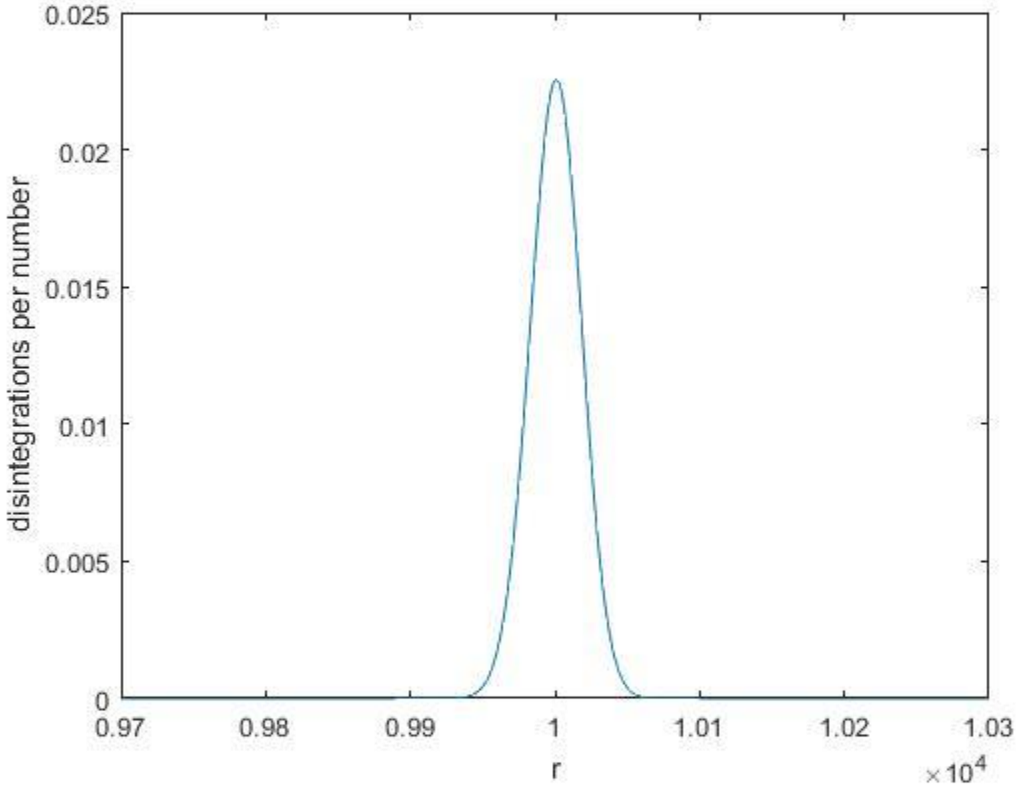

0.025
0.02
0.015
0.01
0.005
0
disintegrations per number
0.97
0.98
0.99
1
1.01
1.02
1.03
r
×10^4

$T = 2\mathbf{t}_{½}$

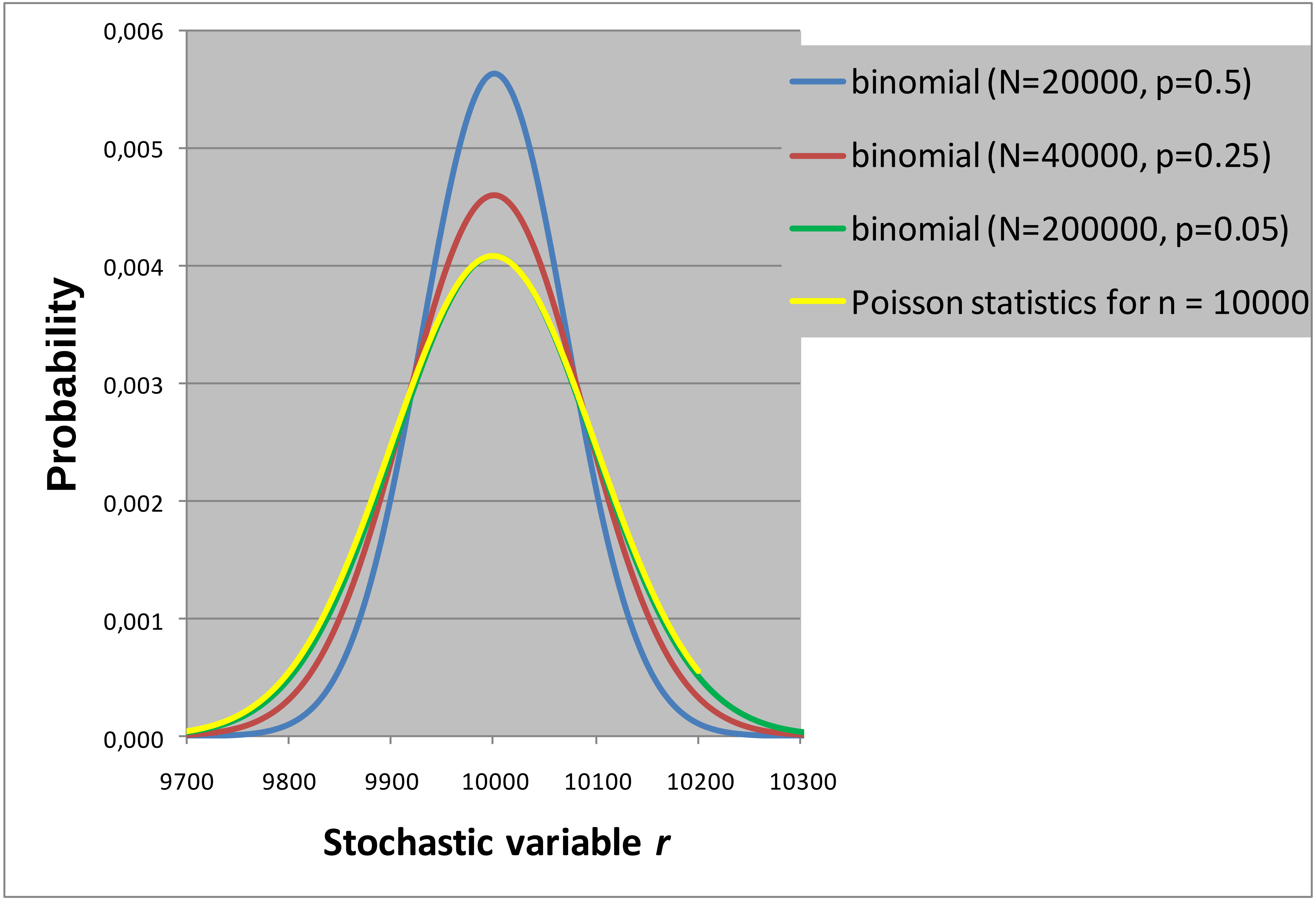


0.025
0.02
0.015
0.01
0.005
0
0.97
0.98
0.99
1
1.01
1.02
1.03
r
×10^4
disintegrations per number

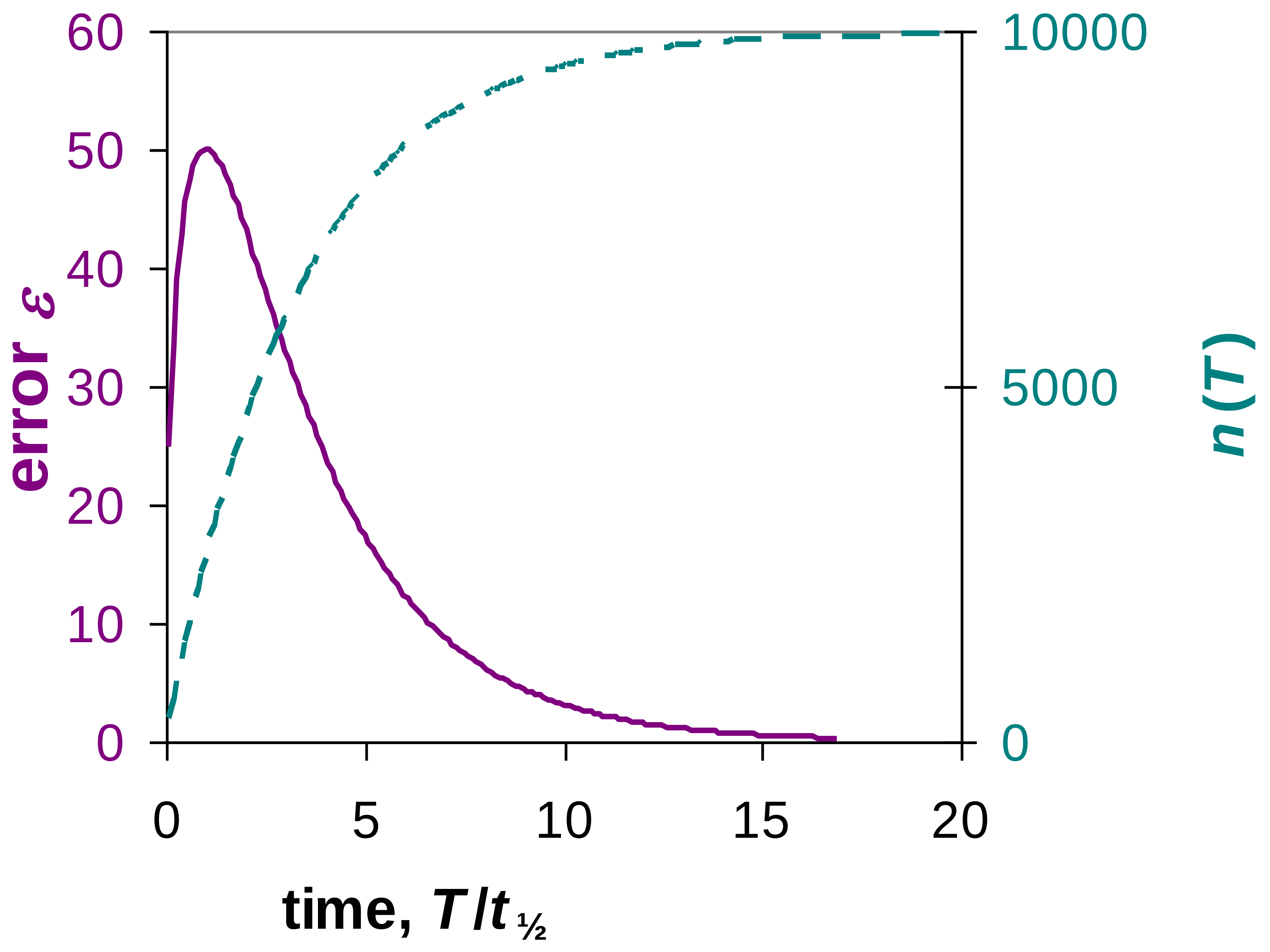
$T = 5\mathbf{t}_{½}$



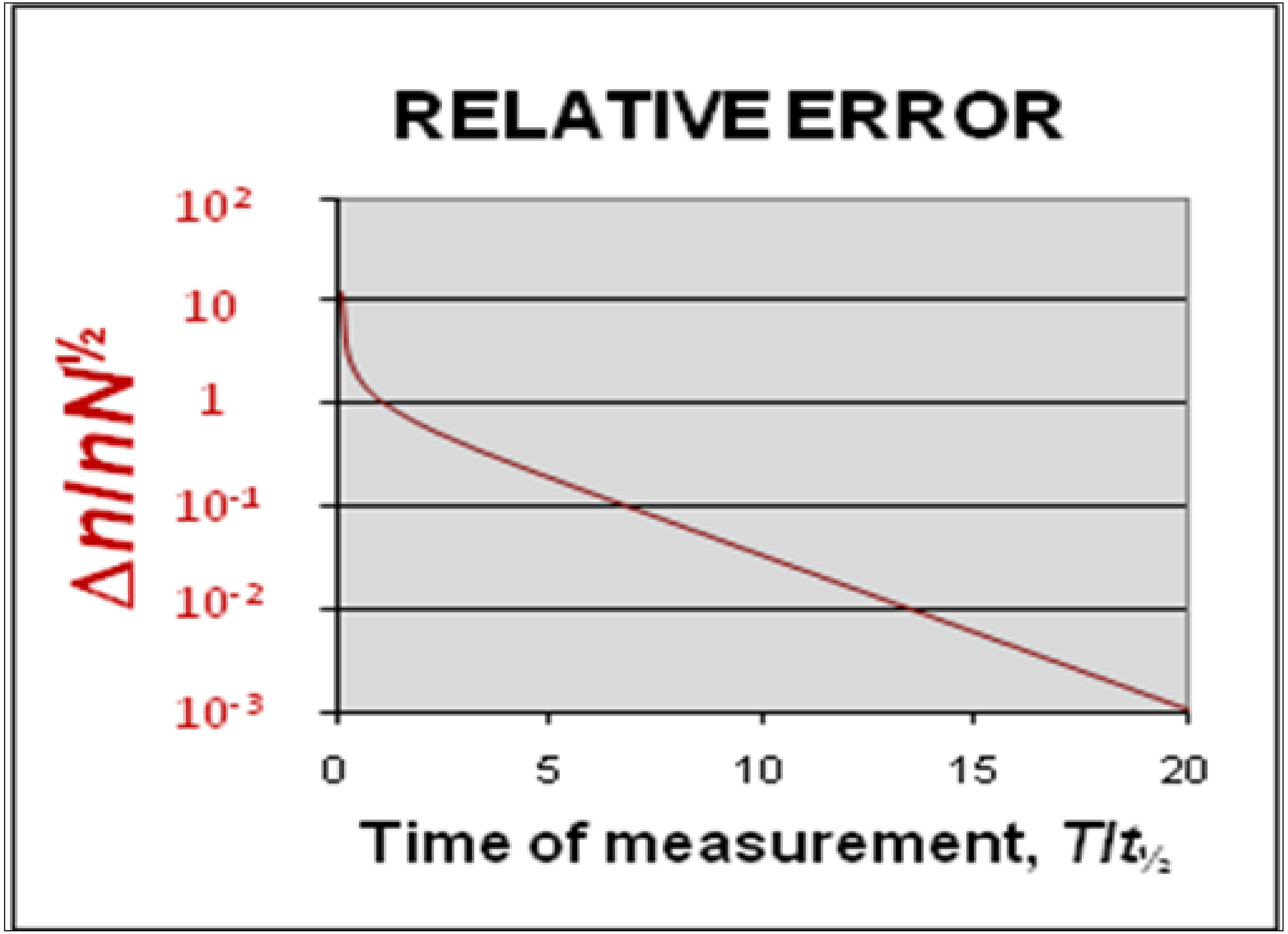
0.025
0.02
0.015
0.01
0.005
0
disintegrations per number
0.97
0.98
0.99
1
1.01
1.02
1.03
r
×10^4



0,006
0,005
0,004
0,003
0,002
0,001
0,000
Probability
9700
9800
9900
10000
10100
10200
10300
Stochastic variable *r*
binomial (N=20000, p=0.5)
binomial (N=40000, p=0.25)
binomial (N=200000, p=0.05)
Poisson statistics for n = 10000

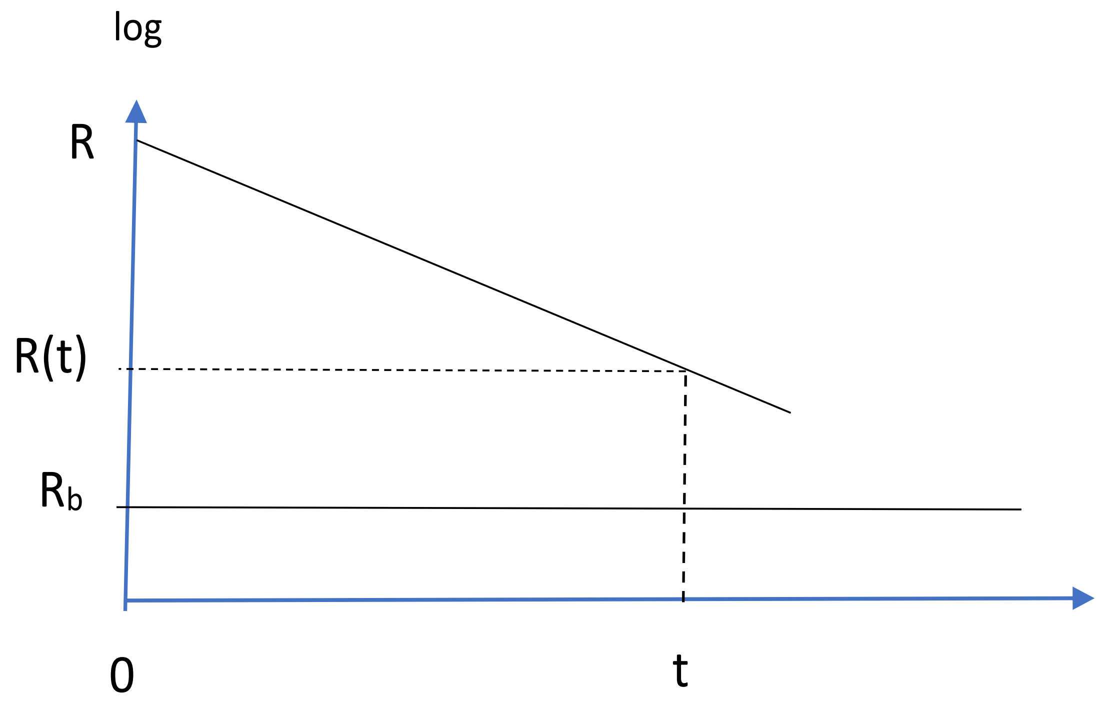


error ε
n(T)
time, T/t½
60
50
40
30
20
10
0
10000
5000
0
0
5
10
15
20



RELATIVE ERROR
10²
10
1
10⁻¹
10⁻²
10⁻³
Δn/nN½
0
5
10
15
20
Time of measurement, T/t½

Counting rate $R = \frac{dn}{dt} = \lambda N \exp(-\lambda T).$



A condition to stop the counting is $\Delta n = R_b T$. Then we find

$$\Delta n = \sqrt{N}\sqrt{1-\exp(-\lambda T)}\exp(-\lambda T/2). = R_b T \therefore \therefore R = \lambda N \frac{R_b^2 T^2}{n}.$$