Stochastics of radioactive decay

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Nuclear disintegration distribution is basically binomial (not Poisson). However, for the binomial distribution the number of nuclides must be known. For short live cases it has been shown that the distribution becomes sharper when the measuring time increases. The accuracy of measurements is considered.

The influence of the background radiation is discussed. A condition is derived for stopping the counting,

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INTRODUCTION

We consider the distribution of radioactive disintegrations. In the radioactive measuring process the distribution of the measured counts can be considerably different from the distribution of disintegrations. The count distribution and the latter are similar in the case of the perfect (4π) detector. The articles [1,2] and this discuss only the distribution of disintegration number.

We consider the well-known equation

$$n = N(1 - \exp(-\lambda T))_i$$

for the number n of disintegrations, where N is the number of identical radioactive nuclides, T is the measuring time and $\lambda = \ln(2)/t_{½}$ is the disintegration constant, and the $t_{½}$ is the half life of the radiation.

DISTRIBUTION OF RADIOACTIVE DISINTEGRATIONS

Set you have N identical nuclides of a radioactive element at the time moment zero. First we consider the disintegration of one nuclide. Fig. 1 is a presentation of that. Its stochastic variable is ν, $\nu \in \{0,1\}$. The event of integration ($\nu = 1$) has the probability $P(1) = p = 1 - \exp(-\lambda T)$ and no disintegration event has the probsbility $P(0)$ = q = 1 – p = exp(- λT). The stochastic process of disintegration of one nuclide is a special case of binomial distributions. Its own name is Bernoulli distribution.

Let us still look Fig. 1 for the disintegration of one nuclide. In this case the expetation (or the mean value) for our stocastic variable ν is

$$E(\nu) = 0\ \mathrm{q} + 1\ \mathrm{p} = \mathrm{p}$$

and the variance

$$Var(\mathrm{v}) = ((0 - E(\mathrm{v}))^2 \mathrm{q} + (1 - E(\mathrm{v})^2 \mathrm{p} = \mathrm{p}^2 \mathrm{q} + (1-\mathrm{p})^2 \mathrm{p} = \mathrm{pq}.$$

For the disintegration of N nuclides we take the stochastic variable $r = \sum_{1}^{N} \mathrm{v}_i$ summing over all N nuclides. $r \in \{0,N\}$ and at the certain value r it has the probability $P(r)$, i.e.

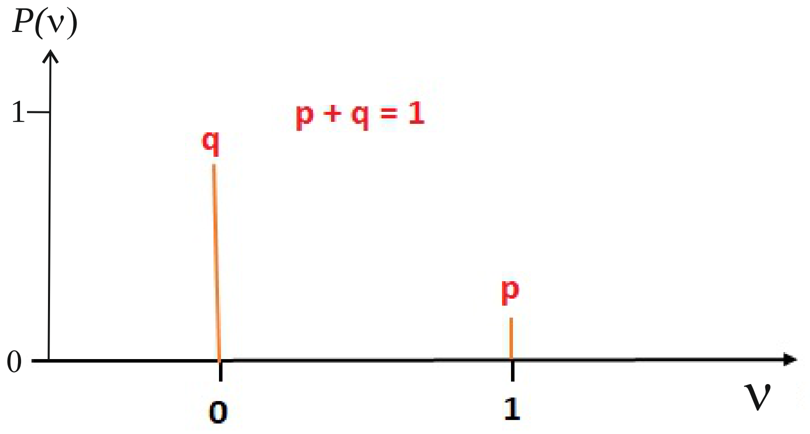


Fig. 1. Stochastic variable ν of one nuclide, binomial distribution of one nuclide or Bernoulli distribution.

$$P(r) = \binom{N}{r} \mathrm{p}^r \mathrm{q}^{N-r}$$
$$= \binom{N}{r} (1 - e^{-\lambda T})^r (e^{-\lambda T})^{N-r}$$

For counting the expectetion $E(r)$ we use the mathematical theorem:

The expectation of the sum of stochastic variables is the sum of the expectations of the variables.

Then $$E(r) = E(\sum_{i=1}^{N} v_i) = \sum_{i=1}^{N} E(\mathrm{v}_i) = N\mathrm{p} = n.$$

For calculation of the variance of *r* we use the theorem

The variance of the sum of independent stochastic variables is the sum of the variances of the variables.

Then (when the nuclides are independent).

$$Var(r) = Var(\sum_{i=1}^{N} v_i) = \sum_{i=1}^{N} Var(\mathrm{v}_i) = N\mathrm{p}\,\mathrm{q} = n \exp(-\lambda T).$$

The standard deviation or error is now

$$\Delta n = \sqrt{n} \exp(-\lambda T / 2). \qquad (1)$$

USE OF BINOMIAL DISTRIBUTION

For calculation binomial distribution the value of *N*, the number of nuclides, is needed. **Note**: it must be an integer. As an example we consider potassium.

Example 1. The radioactive isotope ^{40}K has the half time $t_{½}$ = 1.26 10^9 · It emits the photon 1.460 MeV in 10.72 % emissions. Take a sample of 10 g rock, Make 1 h measurement. Then *N* = 3.96

10^{16}. We find n = 2483 emitted photons per 1 h, and (Eq. 1) Δn = 49.8 and $\Delta n/n$ = 2.0 %. The distribution is in Fig. 2.

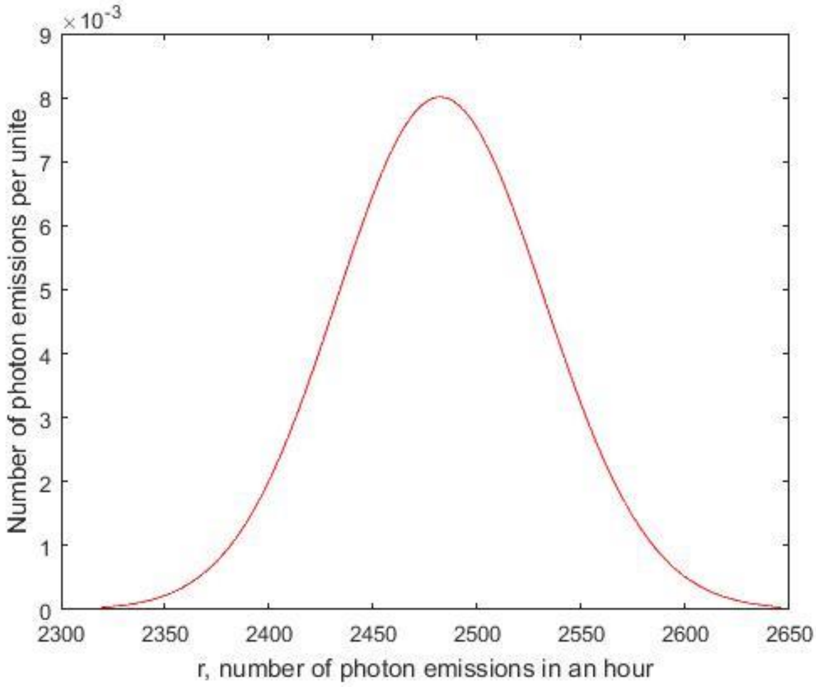


Fig. 2. The distribution of Example 1. This result can be calculated with binomial and with its approximation ($T << t_{½}$) Poisson stochastic model.

In the table 1 the uppermost distribution can be as well calculated by using Poisson model for n = 10 000. In the lower cases *N* is calculated by using *N* = *n*/p, and when necessary the result is given as the closest integer.

The "curves" in Fig. 2 and in the figures of Table 1 illustrate discrete values of *P*(*r*). They obey

$$\sum_{r=0}^{N} P(r) = 1 \text{(binomially)}$$

$$\sum_{r=0}^{\infty} P(r) = 1 \text{ (in the Poisson model).}$$

BACKGROUND CONSIDERATION

In radiation measurement we can follow the counting rate

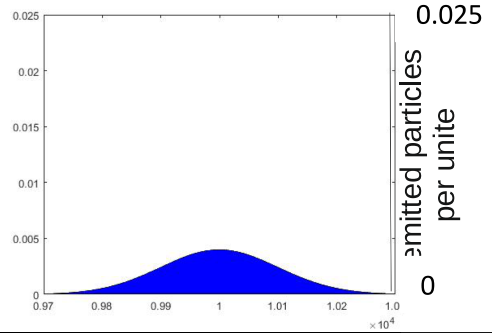
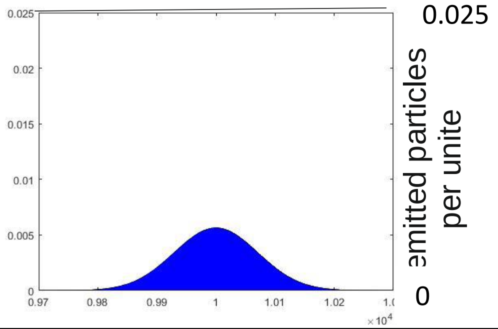
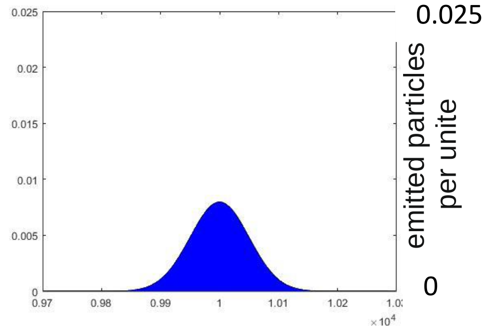
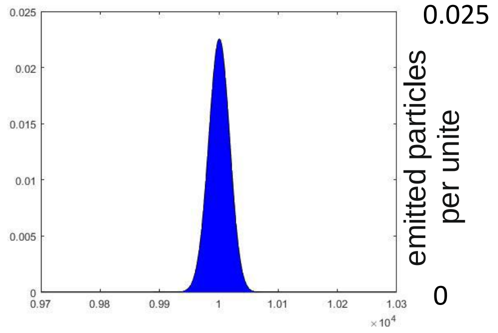
$$R(t) = \frac{dn}{dt} = \lambda p \exp(-\lambda t) = R_0 \exp(-\lambda t).$$

For short live radiation we can set a condition

$$\Delta n = R_b T, \qquad (2)$$

to stop the counting, before the background disturbs. Δn you find from (1). R_b is the background and *T* is the time when in Fig. 3 the counting rate

Table 1. Stochastic distributions $P(r)$ for different T. n = 10 000.

T	p	N	Stochastic distributions $P(r)$
$<<t_{½}$	10^{-4}	100 000 000	
$t_{½}$	$\frac{1}{2}$	20 000	
$2t_{½}$	$\frac{3}{4}$	13 333	
$5t_{½}$	$\frac{31}{32}$	10 323	

$$R(T) = R_0 \frac{R_b^2 T^2}{n} \qquad (3)$$

is met, Eq. (3) has been solved from (2). $R(T)$ increases with T, though n is increasing.

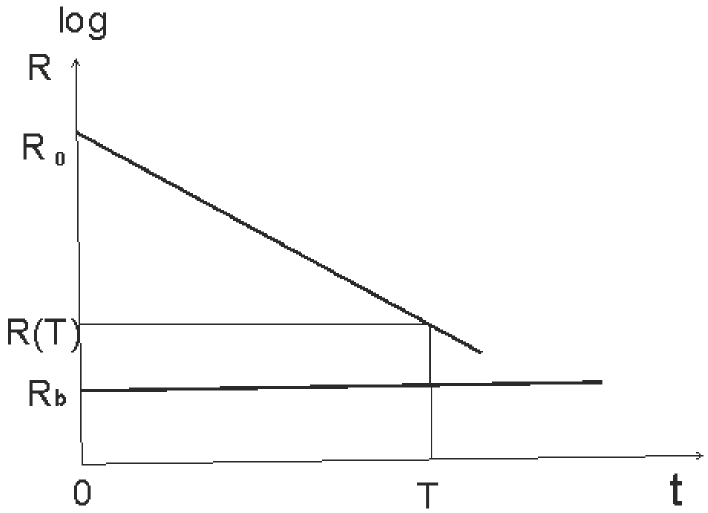


Fig. 3. Counting rate R for a short-lived decay of radioactivity. R_b is the counting rate of background. T is time (Eq. 3) when stop the counting.

CONCLUSION

This paper concerns the emission stochastics of decaying radiation. Each emitting particle (also photon) has similar process to be counted. That side of measurement is permanent. However, when the counting rate is diminishing, then on the emitter side has a change. I think then the Rainwater-Wu [3] idea of binomial model is too simple.

REFERENCES

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